Form Finding of Membrane Shells: Force Density and Isogeometric Methods.

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The state of stress in shell structures can be described by means of stress resultants (or internal forces). Their components can be grouped into three categories, i.e., membrane (or tangential), out-of-plane shear and bending-twisting stress resultants. When the shell structure is constituted by a uniform material, membrane stress resultants are associated with an even distribution of stresses along the transversal direction of the shell, while out-of-plane shear and bending-twisting internal forces cause higher values of stress near the shell mid-surface and at the extremities of the shell thickness, respectively.

An interesting design strategy for shell structures is that of fostering suitable employment of material by determining a structural form such that a specific design load produces only membrane stress resultants and null out-of-plane shear and bending-twisting internal forces. Such a design approach, nowadays referred to as *form- finding* of shells, has a long tradition in the history of structural engineering. It finds practical applications for the design of one- and two- dimensional structural elements having curved geometry and unilateral material constitution, such as masonry arches and vaults, concrete shells, cable networks and fabric tensile structures.

The need of accuracy and efficiency required by the design of the tension structure of the Olympic Park in Munich pushed the development of effective computational form-finding tools. On this occasion, Argyris and coworkers proposed a formulation based on Finite Element Method (FEM) consisting of static analysis of elastic rods in large displacements. Meanwhile, involved in the design of the same structure, Linkwitz and Schek formulated the so-called Force Density Method (FDM). Another family of methods employs the membrane theory of shells. This theory represents a theoretical framework in which the height of the shell mid-surface and the membrane resultant components are related by a unique differential equation.

After introducing the form finding problem and briefly describing the classical force density method, the seminar is mainly focused on a recently formulated form-finding procedure based on Isogeometric Analysis (IgA). This procedure is based on the boundary-value problem described by Pucher's equation. The weak form of that problem is derived and discretized by approximating the relevant fields according to the IgA approach via B-Spline basis functions. The procedure computes a fully compressed state of stress by solving a nonlinear optimization that minimizes the thrusts transmitted to the shell supports. Successively the corresponding shell form is obtained. The procedure is applied to benchmark analytical solution and numerous numerical examples are used to prove the feasibility of the presented approach.