Strengthening of Masonry Arches with Fiber-Reinforced Polymer Strips

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Abstract: This paper deals with masonry arches and vaults strengthened with surface fiber-reinforced polymer (FRP) reinforcement in the form of strips bonded at the extrados and/or intrados, considering strip arrangements that prevent hinged mode failure, so the possible failure modes are: (1) crushing, (2) sliding, (3) debonding, and (4) FRP rupture. Mathematical models are presented for predicting the ultimate load associated with each of such failure modes. This study has shown that the reinforced arch is particularly susceptible to failure by crushing, as a result of an ultimate compressive force being collected by a small fraction of the cross section. Failure by debonding at the intrados may also be an issue, especially in the case of weak masonry blocks or multiring brickwork arches. Failure by sliding has to be considered if the reinforcement is at the extrados and loading is considerably nonsymmetric.

DOI: 10.1061/(ASCE)1090-0268(2004)8:3(191)

CE Database subject headings: Arch; Masonry; Fiber reinforced polymers; Reinforcement; Mathematical models; Crushing; Failure modes.

Introduction

Numerous historical constructions are still in service all over the world and a significant part of them are of cultural and artistic value. Many historical constructions contain masonry arches or barrel vaults, cross or groin vaults, cloist vaults, and domes (i.e., masonry shells), which play an important part in public and residential buildings, as well as in road, rail, and waterway infrastructures.

Modern-day loads are far higher than the ones initially considered. A masonry shell with either tie-rods or nonslender piers only collapses if its loading is severely nonsymmetrical. The safety condition of a common masonry shell consequently depends not on the level of loading, but on the live-to-dead loads ratio. If the ratio is low, modern loads are supported because of the outstanding combination of mechanical properties that masonry shells can rely on to carry symmetrical loads. If the ratio is high (i.e., if today’s live loads are severe), because the live loads distribution may not be symmetrical, modern loads have the potential for causing masonry shells to collapse, in fact, the failure of masonry shells is not unusual.

Substantial alterations often have to be made to masonry buildings to meet present architectural requirements. Such alterations can lead to a significant reduction in the safety margins of masonry shells (e.g., buttresses or tie-rods may be removed, new columns or walls may be rested on the shell’s extrados, openings may be cut, the spandrel fill may be removed or replaced by a lighter material, etc.). As a result, the current usage of many masonry shells either satisfies present needs but fails to fulfill the requirements of modern codes, or it satisfies the codes but is unable to meet the present building, road, rail, or waterway infrastructural demands. Structural engineers consequently often have to assess masonry shells. When the safety margins of a masonry shell are no longer assured or prove inadequate for new demands, then strengthening is needed.

Strengthening masonry shells poses serious concerns because the vast majority is of considerable architectural and historical value. Traditional reinforcement techniques may guarantee an adequate increment in strength, stiffness, and ductility, but are often short-lived and labor-intensive, and they usually violate aesthetic requirements or conservation or restoration needs. Such problems have recently led researchers (Hamid et al. 1994; Modena 1994; Saadatmanesh 1994; Ehsani et al. 1997; Kolsch 1998; Triantafillou 1998a; b; Tinazzi et al. 2000; Albert et al. 2001; Meier 2001; Tong Li et al. 2001; Valluzzi et al. 2001) to suggest strengthening masonry shells with fiber-reinforced polymer (FRP) composites in the form of bonded surface reinforcements.

Reinforcements epoxy-bonded to the masonry surface enable masonry structures to bear substantial tensile stresses, eliminating their greatest mechanical shortcoming at an acceptable cost. Externally bonded reinforcements may be made of steel, but they are much more effective if they are made of FRP. The benefits of FRP over conventional reinforcement materials include its adaptability to curved and rough surfaces, such as historical masonry tends to be.

To ensure adequate masonry permeability and comply with restoration requirements, most of the boundary has to be left without reinforcement. To minimize the amount of FRP while still ensuring an adequate safety margin, reliable methods are needed for the structural analysis of reinforced shells.

The main events leading to the collapse of a masonry shell include severe cracking patterns. Cracking splits the shell into slices, ultimately converting it into a one-dimensional thrusting structure, since the slices behave like arch segments. The ultimate load is therefore carried by a system of masonry arches whose geometry depends on the cracking pattern. Thus the ultimate
behavior of a masonry shell can be simulated by considering a masonry arch.

Reinforcement cannot prevent masonry from cracking. So cracks may form also at a reinforced boundary, but they cannot open because the reinforcement stitches the crack. Thus the FRP reinforcement is incapable of modifying the pattern of cracking in the masonry, and the reinforced shell carries the ultimate load by means of the same system of arches as the unreinforced shell. FRP reinforcement can, however, modify the failure mode of the masonry shell and significantly increase the load-carrying capacity, provided that the reinforcement changes the ultimate behavior of the masonry arches into which the masonry shell converts. The masonry arch simulation considered herein is therefore also suitable for describing reinforced masonry shells, hence the theoretical models presented in this paper can be applied equally well to all masonry shells.

Review of Failure Modes of Masonry Arch

Failure by crushing is unlikely in a masonry arch, since the ultimate normal action provided by the cross section can balance even very severe external loads, i.e., the crushing load greatly exceeds the hinged mode loads (mechanism loads). There are a few exceptions, e.g., an extremely flat arch with tie-rods at the springing, made of poor-quality masonry and loaded symmetrically (in this case, geometrical nonlinearities have to be taken into account too).

Failure by sliding between components could only occur in unrealistically thick arches, but does not happen in real life. In fact, sliding is caused by the excessive inclination of the line of thrust with respect to the cross section. In this case, however, the line of thrust is unable to be defined within the masonry’s thickness and each intersection of the line of thrust with the boundary corresponds to a pin. This means that the sliding load drastically exceeds the hinged mode loads.

So the masonry arch can fail primarily due to a mechanism, consisting of a set of portions of arch joined by pins. Kinematically, a pin behaves like a hinge. Two fundamental differences exist, however, between a pin and a beam-hinge, such as the one used in steel and concrete members, namely: (1) position, and (2) rotation. The pin’s position is on the boundary of the structure, i.e., either at the extrados or intrados of the arch, whereas the position of a beam-hinge is along the axis of the member. The rotation of a pin is unilateral, i.e., only the relative opening of the two pinned sections is possible, whereas both rotations are possible in a beam-hinge. Consequently, the compatibility conditions depend on whether the hinge is on the extrados or intrados. A further difference exists between a pin and a plastic hinge, that is, contrary to the latter, the former does not display any plasticity.

As a consequence of the compatibility conditions, there are two possible shapes for mechanisms, namely (Fig. 1): (1) the arch displacement mechanism (shape 1), and (2) the overall arch-abutment displacement mechanism (shape 3). Shape 2 is unrealistic in practice because for real loads and structures, shape 1 will occur instead. Likewise, shape 4 is insignificant, since it can only result from a lateral movement of the ground or the collapse of a lateral structure in the case of multibay arches.

The flat arch, i.e., the arch with height span-to-rise ratio, exhibits a supplementary shape for mechanisms with respect to the semicircular arch. The extra-shape consists of the reverse of shape 2: the springing hinges are on the intrados, the haunch hinges are on the extrados, and the crown hinge is on the intrados. According to this shape of mechanism, the haunches of the circular arch descend and the crown ascends, while the springing sections do not move.

Collapse Tests on Masonry Prototypes

The experimental procedure consisted of loading full-scale prototypes of brickwork arches, vaults, and domes, with and without various types of FRP reinforcement, up to failure (Figs. 2–11). More than 50 specimens were tested. A complete discussion of the whole experimental program has been presented elsewhere (Faccio et al. 2000; Foraboschi 2001a, b, c; Faccio and Foraboschi 2002), so only a brief overview is provided here.

In Fig. 2, the reinforcement consisted of three 180° intrados strips. The load was applied at a quarter span. After debonding of the reinforcement due to ripping of the brick, the arch failed by the four-hinge mechanism. In Fig. 3, 11 ribbed barrel vaults were tested for three strip arrangements. In all the tests, a quarter-span line-load was applied to the intrados, and the springings were prevented from translating horizontally. The three strip arrangements were as follows. Strip arrangement 1 (two specimens): three 28° strips, 50 mm wide, 2,050 mm spaced, were attached to the intrados of the loading line. The arches failed by the four-hinge mechanism, but the position of the hinges differed from the case of the unreinforced specimen, and the ultimate load was consequently 14.3 times greater. Strip arrangement 2 (two specimens): three 180° strips, 50 mm wide, 2,050 mm spaced, were attached to the whole intrados (outside the ribs). The arches failed by sliding under a load 7.8 times the ultimate load of the unreinforced specimen. Strip arrangement 3 (seven specimens): four 180° strips, 600 mm wide, 1,538 mm spaced, were attached to the whole intrados of three specimens and two 180° strips, 35 mm wide, 3,075 mm spaced, were attached to the whole intrados of four specimens. All the specimens failed by crushing, and all the crushing loads were considerably greater than the hinged mode loads.
In Fig. 4, the specimen was unreinforced. The load was applied to the center of a web. Failure was dictated by the insufficient buttressing action of the webs adjacent to the loaded web. In Figs. 5 and 6, the two specimens were reinforced on 45 and 25% of the extrados, respectively. The load was applied to the center of a web. Failure was dictated by crushing of the loaded web. The ultimate loads were, respectively, 2.5 and 2.1 times the ultimate load of the unreinforced specimen. In Figs. 7 and 8, the specimen was reinforced with an annular strip at the springing. The load was applied at the crown. The dome split into four slices that failed by shape 1 collapse mechanism. Accordingly, the ultimate load was supported without availing of any contribution from the strip. In Fig. 9, the specimen was reinforced with four annular...
strips from the springing to the crown. The strips did not change the dome splitting into slices, but they changed the mode of failure from the hinge mechanism to crushing of the masonry. In Fig. 10, the annular strip did not change the vault splitting into four slices, but it changed the mode of failure from the hinge mechanism to rupture of the FRP in the annular strip, and consequently increased the ultimate load. The annular strip at the haunches (Foraboschi 2001c) proved more effective than the strip at the crown (Fig. 10). In fact, the ultimate load of the specimen with the haunch annular strip was 2.1 times greater than that without. Conversely, the four meridian strips proved useless (the corner ribs were subjected to a prevailing axial force). In Fig. 11, the specimen was reinforced with narrow strips at the intrados. The specimen failed by masonry ripping that caused the strip to become detached.

In the present context, only the following experimental results and interpretations are of interest.

1. Five modes of failure were observed, namely: (1) mechanism (i.e., hinged mode); (2) masonry crushing; (3) sliding (slipping) along a mortar joint; (4) debonding of the FRP reinforcement; and (5) tension rupture of the FRP reinforcement.

2. Each failure by mechanism is due to the hinging behavior of the pins, each located at the boundary opposite the one where the cracks open.

3. A crack still develops even on a reinforced boundary, and therefore also a pin still develops on the opposite boundary, but the reinforcement prevents the hinging behavior of the pin. The hinges of a reinforced structure that fails by mechanism are consequently located in sections different from the endemic hinging sections of the unreinforced structure.

4. Masonry crushing takes place in sections that are cracked, but stitched by the reinforcement. In order to develop a substantial tension force in the strip, the bricks on which the strip is bonded have to rotate rigidly around the boundary opposite the strip. The opposite boundary thus behaves like a pin (but not like a hinge). The pin localizes the contact between the units that rotate around it. Consequently, the stress profile lies at a limited depth, \( y \) (Fig. 12). The pin’s behavior is one of the two reasons why the strengthened arch is particularly susceptible to failure by crushing. Despite the reinforcement, the sections liable to hinging in the unreinforced masonry are still the binding sections, albeit to a considerably lesser degree.

5. Masonry crushing was monitored by numerous strain gauges. In Fig. 13 the compression strains, \( \varepsilon_m \), measured at masonry crushing in two barrel vaults are represented (average \( \varepsilon_m \) of the two specimens). The values of \( \varepsilon_m \) are expressed with respect to the compression strain corresponding to the peak of the masonry constitutive law, \( \varepsilon_{m1} \). The measurements recorded by the strain gauges showed that the portions, \( W \), of masonry in front of the reinforcement strips (Fig. 12) exhibit great compressive strains, while the remaining fraction of the masonry cross section exhibits only marginal compressive strains. Accordingly, the ultimate compressive force is generated by a series of masonry regions, labeled the working area, whose position is in front of the FRP strips [Fig. 12(a)]. The localization of the cross section is the other reason why the reinforced arch is particularly susceptible to failure by crushing.

6. The working area depends on the width and spacing of the strips, as well as on the masonry’s texture, but it is substantially independent of the strip’s axial stiffness or of the masonry’s elasticity modulus. To be more specific, the measurements recorded by the strain gauges show that the boundaries of the working area vary with the material’s properties, but the area they embrace does not (i.e., \( y \cdot W \) is constant). The greater the reinforcement’s axial stiffness with respect to the masonry’s elasticity modulus, the lesser the pin’s rotation; i.e., the greater the depth \( y \) of the working area. Conversely, the greater the reinforcement’s axial stiffness with respect to the masonry’s elasticity modulus, the sharper the cross section’s localization; i.e., the narrower the width \( W \) of the working area. As a result, the working area depends very little on the material’s stiffness, unlike the boundaries of said area.
7. Subsequent processing of the experimental results showed that each working area in front of a strip can be reproduced by a rectangle, whose dimensions [i.e., sides $y$ and $W$, Fig. 12(b)] correspond in width (i.e., $W$) to the blocks (units) to which the strip is bonded plus one block, and in depth (i.e., $y$) to one-third of the thickness. This rectangle reproduces not the shape of the working area, but the area $A_w$ of the surface it embraces.

8. Sliding consists of the slipping of one part in relation to another along a layer of mortar and is due to the inability of the friction mechanism to provide the cross section with the shear action needed to balance the shear demand of the external load. In asymmetrical loading conditions, the cross section most likely to slide is the springing on the less-loaded side.

9. Debonding of the reinforcement consists in either the ripping of a layer of brick or stone, or the detachment of a block from the brickwork (pull-out), while the FRP reinforcement and the bonding layer both remain intact. Debonding only affected reinforcement at the intrados, not at the extrados.

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**Fig. 8.** Failure of the specimen in Fig. 7

**Fig. 9.** Prototype of brickwork dome with four annular reinforcements

**Fig. 10.** Prototype of brickwork cloist vault reinforced by an annular strip at the crown and four meridian strips along the joints of the webs (from the crown to the haunches)
Fig. 11. Detachment of the fiber-reinforced polymer strip due to the ripping of the brick

Fig. 12. (a) Cross section of the reinforced arch (cylindrical vault). The shaded rectangles represent the working area. The width, $W$ (cross-section localization), and the depth, $y$ (pin localization) of each working area is shown. (b) Shaded rectangle (working area) represents the fraction of masonry cross section involved in balancing the tension force in the reinforcement.
10. Strip rupture only occurred when small amounts of FRP were used.

11. In all the tests, crushing, sliding, debonding, and FRP rupture were observed only if the reinforcement prevented all the hinged modes from occurring, whereas the specimens failed by mechanism whenever this possibility remained, even if the hinges were far from their endemic positions. This means that the ultimate loads for crushing, sliding, debonding, and FRP rupture always exceed the ultimate load for the mechanisms.

**Relationships between Fiber-Reinforced Polymer Arrangement and Failure Modes**

The boundary opposite an FRP strip is prevented from hinging. The compatibility conditions (Foraboschi 2000, 2001a) require: (1) the hinges of a mechanism to alternate (extrados-intrados, etc.), and (2) the distance between two consecutive hinges to be greater than the arch’s thickness. Reinforcing the majority (not all) of the extrados or intrados consequently prevents failure by the arch displacement mechanism (shape 1 and 2 of Fig. 1). Reinforcing most of the intrados also prevents the overall arch-abutment displacement mechanism (shape 3 of Fig. 1), whereas this shape of mechanism can still occur even if all of the extrados are reinforced, though this significantly reduces the lateral thrust.

The same results may be achieved by discontinuously reinforcing the boundary, e.g., with two strips on the extrados (on the haunches, maybe) and one on the intrados (on the crown).

The following conclusion can thus be drawn. To assure the masonry shell the greatest possible load-bearing capacity, reinforcement has to prevent all mechanisms from occurring, so that the only feasible types of failure that remain are due to: (1) Crushing, (2) sliding, (3) debonding, or (4) FRP rupture.

When the reinforced arch’s failure is dictated by the mechanism, the ultimate load analysis can be done by adapting the methods typically used for the unreinforced arch. A new version of the lower and upper bound theorem (Foraboschi 2001a) can be used for this purpose. When failure is dictated by a mode other than a mechanism, however, the analysis calls for specific methods (Triantafillou 1998a, b; Valluzzi et al. 2001).

**Failure Analysis by Crushing, Sliding, Debonding, and Fiber-Reinforced Polymer Rupture**

The distribution of the external loads is given. Accordingly, the loading is governed only by a load factor, i.e., the scalar value $q$. To assess the ultimate load factor, $q_u$, hereafter the bending moment is taken with respect to the external reinforcement, instead of the centroid of the cross sections. Let $M'$ be such an internal action. It should be noted that hereafter all the internal actions are expressed per unit of the arch’s width.

**Crushing**

Pin behavior (see experimental results 4) is due to the compressive behavior that is typical of two-dimensional masonry structures. Consider a brickwork panel supported continuously at the base and loaded by a concentrated force at the top (Fig. 14). If the magnitude of the force is low, the force spreads into the panel at a certain angle of diffusion. Diffusion implies a field of tension stresses normal to the compressive stresses (and thus also to the force). If the magnitude of the force is higher, said tension stresses lead to cracks parallel to the compressive stresses, followed by cracks normal to the compressive stresses, and these cracks prevent diffusion. In this latter case, the
force is collected by a vertical strut (chord) just under the force, comprising a limited number of units (i.e., blocks).

To define the number of blocks involved in absorbing the force, let us suppose that the mortar joints consist of beds of unilateral (no-tension) springs that are linear-elastic (in compression), while the bricks consist of rigid blocks. Such an assumption implies (block equilibrium) that the force is collected by a strut with a width equating to three blocks (shaded block, Fig. 14). If the compression constitutive law for the joints allows for a certain plasticity to be displayed, then the width of the strut is even narrower.

The phenomenon described for the masonry panel explains the cross-section localization observed for the test prototypes. The only difference between the reinforced shell and the previous example of a panel is that, for the shell, the phenomenon is not generated by a localization of normal stresses (as it is for the panel), but is the result of tension forces collected by narrow reinforcements with wide gaps between them. The phenomenon is therefore generated by a localization of shear stresses that are balanced by compressive stresses similarly localized in the masonry (as the compressive force of the wall in Fig. 14), and this, together with the pin localization, leads to crushing.

Finally, crushing failure of the reinforced shell is caused and governed by two opposing phenomena, namely, (1) the pin localization, which is due to the bricks’ rotation and governed by the stitching action of the strip; and (2) the cross-section localization, which is due to annular and longitudinal cracking in the masonry (that prevents stress diffusion), and is governed by the interface shear stresses.

Let us consider an arch (i.e., a cylindrical vault) reinforced with a single FRP strip [Figs. 12(b) and 15]. Let S be the thickness of the arch. The cross section consequently has an area equating to $S$ multiplied by the arch’s width (i.e., the length of the generatrices of the cylindrical vault). Let $B$ be the width of the masonry block (brick or stone), and $\beta$ the number of blocks with a strip attached to them. In accordance with the experimental result 7 (justified by previous evaluations), the compressive force localizing the tension force in the FRP strip is collected by a rectangular masonry section—the working area—which area $A_w$ amounts to

$$A_w = \frac{S}{3} \cdot (1 + \beta) \cdot B$$  \hspace{1cm} (1)

According to Eq. (1), therefore, the fraction of the masonry cross section involved in carrying the crushing force can be calculated providing the vault and reinforcement geometry is known. If the reinforcement is attached to both extrados and intrados, the extra-dos reinforcement can be ignored, since the intrados reinforcement alone allows the vault to carry the maximum crushing load.

Let $C$ denote the compressive normal force per unit of width of the arch, acting on a generic cross section. $C$ is generated by the masonry. The ultimate value of $C$ is denoted as $C_u$. Let $\varphi$ be the number of reinforcement strips per unit of the arch’s width and $f_{mc}$ the masonry crushing stress. The experimental stress profiles at crushing exhibit $f_{mc}$ on each area $A_w$ and marginal stresses on the remaining fraction of the cross section. Thus the value of $C_u$ results from $f_{mc}$ uniformly distributed over the working areas. Accordingly, $C_u$ amounts to

$$C_u = \frac{S}{3} \cdot (1 + \beta) \cdot B \cdot \varphi \cdot f_{mc}$$  \hspace{1cm} (2)

Eq. (2) suggests a stress-block profile. Accordingly, $C_u$ is applied at the centroid of $A_w$ (i.e., at $S/6$ from the compressive edge, Fig. 15).

The value of $M'$ in a generic cross section is considered, as well as the related $C$. A linear relationship is hereafter assumed between $q$ and $M'$. This assumption is justified by the fact that the main source of nonlinearity between $q$ and $M'$ is the eccentricity $e' = M'/N$. In the crushing analysis, $e'$ is not high and, above all, it is practically independent of $q$.

Let $M'_{max}$ denote the maximum value of $M'$ in the arch, for a given value of $q$. According to the above hypothesis, $M'_{max} = K \cdot q$, where the dimensional constant $K$ depends only on the arch, and so it is known. Thus

$$q = (1/K) \cdot M'_{max}$$  \hspace{1cm} (3)

The moment taken with respect to the reinforcement (i.e., to the edge of the cross section) gives the following relationship between $C$ and $M'_{max}$ (Fig. 15):

$$\frac{5}{6} \cdot S \cdot C = M'_{max}$$  \hspace{1cm} (4)

Eq. (4), in conjunction with Eq. (3), gives the relationship between the crushing load factor, $q_{uc}$, and $C_u$:

$$q_{uc} = \frac{\frac{5}{6} \cdot S \cdot C_u}{K}$$  \hspace{1cm} (5)

Substituting Eq. (2) into Eq. (5) gives us $q_{uc}$

$$q_{uc} = \frac{\pi \cdot (1 + \beta) \cdot B \cdot \varphi \cdot f_{mc} \cdot S^2}{K}$$  \hspace{1cm} (6)

Eq. (6) solves the problem. The value of $q_{uc}$ depends on the magnitude and point of application of $C_u$. The magnitude of $C_u$
results from the assumption of a stress-block distribution on $A_w$. The deviations between the experimental and assumed values of $A_w$ are marginal. Moreover, the observed strains are very high inside $A_w$, but very low outside (Fig. 13), so only marginal deviations between the actual and assumed magnitude of $C_u$. The point of application of $C_u$ stems from the depth, $y$, of $A_w$. The model assumes $y = S/3$. The deviations between the experimental and assumed values of $y$ are negligible. A sensitivity analysis of $q_{uc}$ on $y$ was developed. Since $K$ does not depend on $y$, $q_{uc}$ can be replaced by $M_{max}$. The general form of $M_{max}$ is

$$M_{max} = C_u \left( S - \frac{y}{3} \right)$$

The ratio of the first-order term to the zero-order term of the expansion of $M_{max}$ in a Taylor series about the point $y = S/3$ gives us

$$\frac{I \text{ order}}{0 \text{ order}} = \frac{y - S}{3} = \frac{3}{10} \left( y - \frac{S}{3} \right) = \frac{3}{10} \left( \frac{y}{S} - \frac{1}{3} \right)$$

The maximum value of $y$ observed in the tests was $S/2$ and the minimum was $S/3$. Accordingly, the greatest deviation of $y$ with respect to $S/3$ was $\left[ 3 \cdot S/39 - 13 \cdot S/39 \right] = 10 \cdot S/39$. Thus $I \text{ order/0 order} = 7.7\%$. As a result, the error on $q_{uc}$ is no more than $8\%$, and is therefore acceptable. The above result supports the following extrapolation. If $M' > 0$ (i.e., if the strip has to have a tensile force), the distance between the point of application of $C$ and the compressive edge can be adequately reproduced by the value $S/6$ even if $\sigma < f_{mc}$.

To illustrate the applicability and practicality of the proposed analysis, the model was applied (Fig. 16) to the specimens with the third strip arrangement of Fig. 3 (i.e., 180° intrados strips) that failed by crushing. The 600 mm wide strips were attached to three bricks ($1 + \beta = 4; \varphi = 0.65$), and the 35 mm wide strips to one brick ($1 + \beta = 2; \varphi = 0.33$). The experimental value of $f_{mc}$ was 7.35 N/mm². The application of Eq. (2) to the two strip arrangements gives us $C_u = (124/3) \cdot 4 \cdot 245 \cdot 0.65 \cdot 7.35 = 193.5$ kN and $C_u = 49.1$ kN, respectively. Eq. (4) written for $C_u$ gives us $M'_{max} = 20.00$ kN m and $M'_{max} = 5.08$ kN m, respectively. Since the extrados was not strengthened, at crushing the structure exhibited three intrados hinges. Consequently, the crushing load and internal actions were only correlated by considering equilibrium (three-hinged arch). The experimental values of $M'_{max}$ can therefore be assessed directly from the experimental crushing loads. The two values of $M'_{max}$ (i.e., the average values of the two sets of specimens) were 23.20 and 5.72 kN m, respectively. The average deviations between the calculated and measured values were consequently no more than 14%, which is an acceptable value.

### Sliding

The reinforced masonry withstands the shear action in the same way as its unreinforced counterpart, by means of Coulomb’s friction law, according to the masonry friction coefficient $\mu \approx 0.5$ (Heyman 1982). But the resisting friction shear is generated by
the resultant of the compressive stresses on the masonry cross section, C, not by the normal internal action N (Fig. 17).

Sliding is the result of an excessive shear action V. The ultimate value of V, V_u, again expressed per unit of width, is therefore (Fig. 17)

\[ V_u = C \cdot \mu. \tag{9} \]

Sliding load causes the reinforcement to elongate. Consequently, the distance between C and the compressive edge is still S/6. Using the cross-sectional equilibrium, C can therefore be expressed as a function of \( M' \) (Fig. 15). Replacing C with said function in Eq. (9) yields

\[ V_u = \mu \cdot 1.2 \cdot M'/S \tag{10} \]

The sliding load factor is the lowest \( q \) for which \( V = V_u \) in a generic cross section. It is essential to note that \( V_u \) also depends on the thickness S. To design reinforcement to prevent sliding, \( M' \) can be controlled by the arrangement of the FRP strips.

The proposed analysis was applied to the specimens with the second strip arrangement of Fig. 3 (i.e., extrados strips), that failed by sliding (Fig. 16). The analytical value of the sliding load coincides with the average experimental values for \( \mu = 0.64 \). Such a value of \( \mu \) is more realistic than \( \mu = 0.5 \), the latter being conservative.

**Debonding**

Basic theoretical evaluation proves that a reinforcement strip epoxy-bonded onto a curved surface involves a significant concentration of normal stresses, \( \sigma \), transverse (normal) to the bonding masonry boundary, denoted as \( \sigma_{\perp} \), along with the shear stresses, \( \tau \), parallel to the bonding masonry boundary. In fact (Fig. 18), the transverse equilibrium of the infinitesimal arc of reinforcement around a masonry crack reveals that \( \tau \) are balanced tangentially, but not transversally (radially). Only a transfer of \( \sigma_{\perp} \) between reinforcement and masonry can balance the \( \tau \) transfer. The thickness of the reinforcement is disregarded here, so the following relationship relates \( \sigma_{\perp} \) with the unit tension force in the reinforcement, T

\[ \sigma_{\perp} = \frac{T}{(\lambda \cdot \varphi \cdot F)} \tag{11} \]

in which \( \lambda \) denotes the radius of curvature and \( F \) = width of the strip that exhibits \( T \).

Eq. (11) shows that, since the \( \sigma_{\perp} \) are due to the curvature, they exist even if the reinforcement has a thickness of nil and can be evaluated independently of the \( \tau \). Eq. (11) also shows that intrados reinforcement implies +\( \sigma_{\perp} \) (tension), while extrados reinforcement means −\( \sigma_{\perp} \) (compression). While debonding in composite-reinforced concrete beams can also result from \( \tau \) stresses, in masonry shells the other than debonding modes of failure induce very low \( \tau \) stresses that are far below the ultimate limit of \( \tau \), which corresponds to a much higher ultimate load than for the other failure modes. Debonding analysis will consequently only be concerned with intrados reinforcement.

It can be stated beforehand that the \( \sigma_{\perp} \) only exist on the bricks adjacent to cracks, so the actually or potentially cracked sections, i.e., the cross sections where \( T > 0 \), are considered here (i.e., \( M' > 0 \) and therefore the point of application of C is still that of Fig. 15). The relationship that links \( T \) with \( N \) and \( M' \) is (Fig. 15)

\[ T = \frac{1.2 \cdot M'}{S} - N \tag{12} \]

If Eq. (12) is used in Eq. (11) for \( T \), then \( \sigma_{\perp} \) can be calculated as

\[ \sigma_{\perp} = \frac{1.2 \cdot M'}{S} - N \frac{1}{(\lambda \cdot \varphi \cdot F)} \tag{13} \]

However, the crucial point is to establish the limit of the \( \sigma_{\perp} \). For this purpose, the theoretical analysis has fully incorporated the experimental results. The \( \sigma_{\perp} \) are collected by the epoxy matrix, not by the fibers, but the experiments demonstrated that the FRP composite as well as the epoxy layer always remain intact. It can thus be stated that, if the FRP reinforcement is properly applied, the epoxy resin is stronger than the masonry under tension, so failure is always dictated by the masonry and not by the reinforcement. The experiments indicated two failure modes for debonding, namely: (block) ripping and (block) pull-out.

Considering the block ripping, the experimental evidence shows that the transversal tensile strength of the brick, denoted \( \sigma_{\ast} \), is the binding quantity, since this failure mode consists of the ripping of a 3–6 mm brick layer. \( \sigma_{\ast} \) can be measured by in situ tests or obtained from the technical literature. Failure occurs once \( \sigma_{\perp} \) reaches \( \sigma_{\ast} \) in a section.

If the angle formed by a cross section with the horizontal is denoted as \( \theta \), and \( \theta_{\ast} \) = angle defining the ripping section (if the loading is symmetric, the \( \theta \)’s are two) obtained by solving the following equation, in which \( M' \) and \( N \) are functions of \( \theta \) and \( q \), and \( S \), \( \lambda \), \( \varphi \), and \( F \) are constants:

\[ \frac{\partial}{\partial \theta} \left( \frac{1.2 \cdot M'(\theta; q)}{S} - N(\theta; q) \right) = 0 \tag{14} \]

Eq. (14) gives us the following expression for the ripping load-factor \( q_{\ast} \):
\[ q_{ur} = \frac{\lambda \cdot \varphi \cdot F \cdot \sigma_{mat}^{\mu}}{1.2 \cdot M'(\theta_m:q = 1) \cdot S} - N(\theta_m:q = 1) \]  \hspace{1cm} \text{(15)}

Eq. (15) is based on the linearity between \( q \) and \( M' \).

Considering debonding by block pull-out, we let the transversal force \( R_{\perp} \) be the resultant of the friction between each of the two radial faces of the brick and the adjoining faces, since (with the exception of multiring brickwork vaults) at least half of the blocks cross the whole thickness. Friction is triggered by the resultant of the compressive stresses, \( C \), along with the friction coefficient. For the latter, a value of 0.5 may be assumed (Heyman 1982). For the former (Fig. 15), the relation \( C = 1.2 \cdot M'/S \) can be used. Thus \( R_{\perp}^u \) is

\[ R_{\perp}^u = 2 \cdot \mu \cdot 1.2 \cdot M'(\theta)/S = 2.4 \cdot \mu \cdot M'(\theta)/S \]  \hspace{1cm} \text{(17)}

\( R_{\perp}^u \) turns out to be a function of \( \theta \) and \( q \), as well as \( R_{\perp} \). The lower \( q \) for which \( R_{\perp} \) equals \( R_{\perp}^u \) for a generic \( \theta \), i.e., for which the right-hand terms in Eqs. (16) and (17) are equal at a \( \theta \), is the pull-out load-factor.

The proposed analysis was applied to the arch bridge of Fig. 2 that failed by block ripping. Tests provided \( \sigma_{mat}^{\mu} = 0.54 \ N/mm^2 \). Eq. (11) provides the maximum tension force in the unitary reinforcement (\( \varphi = 1.06 \)); \( T = 0.54 \cdot 55 \cdot 1.06 \cdot 5.015/2 = 78.81 \ kN/m \). Strain gauges were placed on the FRP reinforcement. The measurements recorded at the time of ripping show \( T = 98.59 \ kN/m \).

The error of the model was 20%. Thus the same error is present in predicting the debonding load (at debonding, the structure was statically determinate). This error, albeit acceptable, is due to the fact that masonry under tension allows for a certain plasticity to be displayed. To refine the model, debonding has to be analyzed within the framework of fracture mechanics, following, for example, the recent works of Boyajian et al. (2002a, b).

**Fiber-Reinforced Polymer Tension Rupture**

Let \( T_{FP} \) be the ultimate axial tension force in the reinforcement per unit width of strip. FRP tension rupture occurs at the lowest \( q \) for which the following equation is verified in a generic cross section where \( M'(\theta) > 0 \) (Fig. 15)

\[ 1.2 \cdot M'(\theta)/S = T_{FP} \cdot F \cdot \varphi \]  \hspace{1cm} \text{(18)}

**Conclusions**

The failure modes of a masonry arch whose hinged mode failures (mechanisms) are prevented by FRP reinforcement were analyzed to obtain the ultimate load for each mode of failure, the lowest of which constitutes the strength of the reinforced masonry arch.

There is a further, fundamental problem to solve. While a mechanism is implicitly determined statically, this may not be true of failure modes other than mechanisms. While in the former the springing thrust, and therefore also \( M' \) and \( N \), are known directly, in the latter the internal actions cannot be known without considering compatibility and constitutive laws. This problem involves a low-strength material analysis on two-dimensional structures that can yield only a rough solution, which also varies considerably in time (with creep, cracking, etc.). If the ultimate load is dictated by a mode of failure other than mechanisms, a series of hinges develops opposite the unreinforced boundaries. In many cases, the hinges enable the structure to be statically determined. Otherwise, it is advisable to use the lower thrust of the arch. The springing thrust is known to exceed said lower value, which depends on the geometry and the loading alone (Blasi and Foraboschi 1994; Foraboschi and Blasi 1996). The lower thrust of the reinforced arch can be calculated using the method proposed by Foraboschi (2001a).

Parametric analyses have shown that the load-bearing capacity can be increased significantly using a small quantity of surface FRP reinforcement, but the entity of said increment depends largely on the arrangement of the reinforcement. It is not only the location of the reinforcement that influences strength, but also the spacing and the width of the strips, or the number of blocks to which a strip is bonded. Finally, to prevent the collapse mechanism by bonding external FRP reinforcement and consequently force the arch to fail by other less critical modes, lends the structure: (1) a significant increase in load-bearing capacity, greater than for the hinging modes; (2) an appreciable reduction in lateral thrust, and (3) a more certain and predictable ultimate behavior.

This research will be further developed to consider debonding using the tools of fracture mechanics. Although the precision achieved by the model proposed here seems to be adequate for design purposes, a refinement of the formulation would be valuable.

**Notation**

The following symbols are used in this paper:

- \( A_w \) = area of the working area (m²);
- \( B \) = width of the block (masonry unit, i.e., brick or stone) (m);
- \( C \) = unitary compressive normal force internally acting on a masonry cross section (N/m);
- \( C_u \) = crushing normal action of a masonry cross section (N/m);
- \( f_{mc} \) = masonry crushing stress (N/m²);
- \( M' \) = unitary bending action with respect to the reinforced boundary (N/m/m);
- \( M'_{\max} \) = maximum \( M' \) in the arch [i.e., maximum of \( M'(\theta) \)] (N/m/m);
- \( N \) = unitary normal action (N/m);
- \( q \) = load factor;
- \( q_{uc} \) = crushing load factor;
- \( q_{ur} \) = ripping load factor;
- \( R_{\perp} \) = transversal force applied to a brick (N);
- \( R_{\perp}^u \) = ultimate value of \( R_{\perp} \) (N);
- \( S \) = thickness of the arch (m);
- \( T \) = tension force in the reinforcement for an unitary width of the fiber-reinforced polymer strip (N/m);
- \( T_{FP} \) = ultimate T (N/m);
- \( V \) = shear action in an unitary cross section (N/m);
- \( V_u \) = unitary ultimate value of the shear action V (N/m);
- \( W \) = width of \( A_w \) (m);
\[ y = \text{depth of } A_w \ (m); \]
\[ Z = \text{length of the block, i.e., annular dimension (m)}; \]
\[ \beta = \text{number of blocks with a strip attached to them}; \]
\[ \varepsilon_m = \text{compression strain measured in the masonry, at the crushing of the vault}; \]
\[ \varepsilon_{m1} = \text{compression strain corresponding to the peak of the masonry stresses}; \]
\[ \theta = \text{angle formed by a cross section with the horizontal (rad)}; \]
\[ \theta_m = \text{rippling section (rad)}; \]
\[ \lambda = \text{radius of curvature of the intrados of the arch (m)}; \]
\[ \mu = \text{masonry friction coefficient (—)}; \]
\[ \sigma = \text{normal stress (N/m}^2); \]
\[ \sigma_{t1} = \text{stresses transverse to the bonding surface (N/m}^2); \]
\[ \sigma_{t2} = \text{transversal tensile strength of the brick (N/m}^2); \]
\[ \tau = \text{tangential stress on the bonding masonry boundary, the bonding layer, and the strip (N/m}^2); \]
\[ \varphi = \text{number of fiber-reinforced polymer strip reinforcements per unit of the arch’s width}. \]

References


